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Quantum 1/f noise is a basic property of physical cross sections and process rates and a form of quantum chaos in the nonlinear system of the charged particles plus the electromagnetic field. Therefore the present report starts with a consideration of the general problem of 1/f spectra in nonlinear systems, derives for the first time a general sufficient criterion which tells us if a system will show 1/f noise, and applies the new criterion to transport in semiconductors, in metals, on highways, and in quantum electrodynamics. In all these cases 1/f spectra follow from the same criterion, in the same way. This is, for the first time, a unifying principle. In addition, the report contains the first rigorous first principles derivation of quantum 1/f mobility fluctuations in semiconducting materials (analytical) and reference to a Monte Carlo simulation of the same problem. Finally, a solution for the long-standing problem of quantum 1/f noise in the collector of BJT's is proposed.

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QUANTUM 1/f NOISE IN HIGH TECHNOLOGY APPLICATIONS INCLUDING ULTRASMALL STRUCTURES AND DEVICES

FIRST ANNUAL REPORT

July 15, 1989 - July 14, 1990

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Abstract

The fundamental fluctuations of physical cross sections and process rates in Quantum Electrodynamics are known as quantum 1/f effect, are limiting most high-technology applications, and are described by the simple quantum 1/f formula for the fractional spectral density, $S(f)=\alpha A/fN$. In spite of their basic character, they are a special case of 1/f chaos in nonlinear systems. The present report provides for the first time a sufficient criterion for the presence of a 1/f spectrum in a nonlinear system, proves its sufficient character, and exemplifies it for the current carriers in semiconductors, for electrons in metals, for cars on the highway, and for the nonlinear system of particle and field in electrodynamics. The report gives some of the rigorous first principles analytical calculation results for the quantum 1/f mobility fluctuations in electronic materials such as Si, GaAs, $Hg_{1-x}Cd_xTe$, etc., derived for the first time here under the present grant, on the basis of the quantum 1/f cross correlation formulas derived under the previous AFOSR Grant 85-0130. These quantum 1/f results give the conventional quantum 1/f noise as an analytical function of temperature, applied field, and all the physical parameters of the semiconductor material, allowing for optimization of the material for any given application. For the quantum 1/f noise in $Hg_{1-x}Cd_xTe$ we also developed a Monte Carlo computer simulation, which is currently being tested and improved, and which will be compared with results calculated analytically. Finally, a suggestion is presented, which provides an explanation of the discrepancies which have been noticed in the collector current 1/f noise of bipolar junction transistors.

CONTENTS

I. INTRODUCTION

Progress has been achieved during 1989 and 1990 in the study of nonlinear systems which generate chaotic 1/f fluctuations, in the application of the Quantum 1/f Theory[1-3] to various materials used in small and ultrasmall electronic devices, and in the application of the Quantum 1/f Theory to electronic devices.

The close similarity of the classical and quantum 1/f theories, and the initial development of the quantum 1/f theory by the author out of his efforts to quantize his classical turbulence theory, have led to sustained efforts of the author aimed to integrate all his theories as various forms or realizations of a fundamental notion of chaos in nonlinear systems. During this grant period, these efforts finally beared fruits. A general sufficient criterion was formulated, allowing to identify the nonlinear systems which exhibit 1/f spectra. This criterion is presented for the first time in Sec. II below. It is followed in Sec. III by examples, in which the criterion is applied to the classical and quantum mechanical forms of the author's 1/f noise theory. These examples clarify the physical meaning of the new criterion.

For the practical application of the Quantum 1/f Theory it is necessary to derive the quantum 1/f fluctuations of various kinetic (transport) coefficients which characterize the materials used in electronic and microelectronic applications, from the author's fundamental quantum 1/f formula. The latter is applicable only to cross sections and rates of elementary processes. Most important is the calculation of mobility fluctuations in Si, GaAs and $Hg_{1-x}Cd_xTe$. An earlier calculation (Kousik, Van Vliet, Handel, 1985) of mobility fluctuations in Si and GaAs is replaced in Sec. IV by a more rigorous calculation, based on the new quantum 1/f cross-correlations, developed under the previous AFOSR Grant, and presented in the Final Technical Report AFOSR -85-0130. The new calculation yields increased 1/f noise, and is in vey good agreement with the experiment. Although not presented here yet, we also performed a Monte Carlo simulation for HgCdTe during this period. The simulation has yet to be improved and compared with the experiment.

Finally, we have tried to improve the application of quantum 1/f theory to the collector noise of bipolar transistors. This short calculation is presented in Sec. V.

In the same time, many new contributions to the quantum 1/f theory and experiment were published by other workers in the field, considerably advancing the field of infra-quantum physics and quantum 1/f noise in high-technology applications. These new contributions, as well as new PhD thesis work in this field and contributions presented at the IV Conference on Quantum 1/f Noise and other Low-Frequency Fluctuations, are included in the updated General Quantum 1/f Bibliography appended to this Report.

II. A SUFFICIENT CRITERION FOR 1/F NOISE IN NONLINEAR SYSTEMS

Consider a n-dimensional nonlinear system described in terms of the dimensionless function $\mathbf{Y}(\mathbf{x},t)$ by the m^{th} order nonlinear dynamical equation

$$d\mathbf{Y}/dt + F(\mathbf{x}, \mathbf{Y}, d\mathbf{Y}/dx_1 ... d\mathbf{Y}/dx_n, d^2\mathbf{Y}/dx_1{}^2 ..... d^m\mathbf{Y}/dx_n{}^m) = 0 \qquad (1)$$

If

$$F[\lambda\mathbf{x}, \mathbf{Y}, d\mathbf{Y}/(\lambda dx_1) ... d\mathbf{Y}/(\lambda dx_n), d^2\mathbf{Y}/(\lambda dx_1)^2 ..... d^m\mathbf{Y}/(\lambda dx_n)^m] = \lambda^p F(\mathbf{x}, \mathbf{Y}, d\mathbf{Y}/dx_1 ... d\mathbf{Y}/dx_n, d^2\mathbf{Y}/dx_1{}^2 ..... d^m\mathbf{Y}/dx_n{}^m) \qquad (2)$$

for any real number λ, Eq. (1) is said to be homogeneous. Performing a Fourier transformation with respect to the vector $\mathbf{x}$, we get in terms of the Fourier-transformed wavevector $\mathbf{k}$ the nonlinear integro-differential equation

$$d\mathbf{y}(\mathbf{k},t)/dt + G[\mathbf{k}, \mathbf{y}(\mathbf{k},t), k_1\mathbf{y}(\mathbf{k},t) ... k_n\mathbf{y}(\mathbf{k},t), k_1{}^2\mathbf{y}(\mathbf{k},t) ..... k_n{}^m\mathbf{y}(\mathbf{k},t)] = 0, \qquad (3)$$

where $\mathbf{y}(\mathbf{k},t)$ is the Fourier transform of $\mathbf{Y}(\mathbf{x},t)$. Due to Eq. (2), G satisfies the relation

$$G[\lambda \mathbf{k}, \mathbf{y}, \lambda k_1 \mathbf{y} \ldots \lambda k_n \mathbf{y}, (\lambda k_1)^2 \mathbf{y} \ldots\ldots (\lambda k_n)^m \mathbf{y}]$$
$$= \lambda^p G[\mathbf{k}, \mathbf{y}, k_1 \mathbf{y} \ldots k_n \mathbf{y}, k_1{}^2 \mathbf{y} \ldots\ldots k_n{}^m \mathbf{y}]. \quad (4)$$

Eq. (3) can thus be rewritten in the form

$$d\mathbf{y}/d(t/\lambda^p) + G[\lambda \mathbf{k}, \mathbf{y}, \lambda k_1 \mathbf{y} \ldots \lambda k_n \mathbf{y}, (\lambda k_1)^2 \mathbf{y} \ldots\ldots (\lambda k_n)^m \mathbf{y}] = 0, \quad (5)$$

Taking $\lambda=1/k$, where $k=|\mathbf{k}|=(x_1{}^2+\ldots.+x_n{}^2)^{1/2}$, and setting $k^p t=z$, we notice that k has been eliminated from the dynamical equation, and only $\mathbf{k}/k$ is left. This means that there is no privileged scale left for the system in $\mathbf{x}$ or $\mathbf{k}$ space, other than the scale defined by the given time t, and expressed by the dependence on z. We call this property of the dynamical system "*sliding-scale invariance*".

In certain conditions, instabilities of a solution of Eq. (1) may generate chaos, or turbulence. In a sufficiently large system described by the local dynamical equation (1), in which the boundary conditions become immaterial, homogeneous, isotropic turbulence, (chaos) can be obtained, with a spectral density determined only by Eq. (1). The stationary autocorrelation function $A(\tau)$ is defined as an average over the turbulent ensemble

$$A(\tau) = \langle \mathbf{Y}(\mathbf{x},t)\mathbf{Y}(\mathbf{x},t+\tau)\rangle = \int \langle \mathbf{y}(\mathbf{k},t)\mathbf{y}(\mathbf{k},t+\tau)\rangle d^n k = \int u(k,z) d^n k \quad (6)$$

Here we have introduced the scalar

$$u(k,z) = \langle \mathbf{y}(\mathbf{k},t)\mathbf{y}(\mathbf{k},t+\tau)\rangle \quad (7)$$

of homogeneous, isotropic chaos (turbulence), which depends only on $|\mathbf{k}|$ and $z=k^p\tau$. All integrals are from minus infinity to plus infinity. The chain of integro-differential equations for the correlation functions of any order obeys the same sliding-scale invariance which we have noticed in the fundamental dynamical equation above. *Therefore, in isotropic, homogeneous, conditions, u can only depend on k and z.* Furthermore, the

direct dependence on k must reflect this sliding-scale invariance, and is therefore of the form

$$u(k,z) = k^{-n}v(z). \tag{8}$$

Indeed, only this form insures that $u(k,z)d^nk$ and therefore also the corresponding integrals and multiple convolutions in k space have the necessary sliding-scale invariance.

. According to the Wiener-Khintchine theorem, the spectral density is the Fourier-transform of A(t),

$$S_y(f) = \int e^{2\pi i f\tau}A(\tau)d\tau = (1/f) \int e^{2\pi i t'}\int k'^{-n}v(z)d^nk'dt' = C/f, \tag{9}$$

where we have set $f\tau=t'$, $k^n=fk'^n$, $z=k^n\tau=k'^nt'$, and the integral

$$C = \int e^{2\pi i t'}\int k'^{-n}v(z)d^nk'dt' = \int e^{2\pi i t'}\int k''^{-n}v(k''^n)d^nk''dt' \tag{10}$$

is independent of f. We have defined the vector $k''=t'^{1/n}\,\mathbf{k}$.

In conclusion, we have shown that if the equation

$$d\mathbf{Y}/dt + F(\mathbf{x}, \mathbf{Y}, d\mathbf{Y}/dx_1...d\mathbf{Y}/dx_n, d^2\mathbf{Y}/dx_1{}^2.....d^m\mathbf{Y}/dx_n{}^m) = 0 \tag{11}$$

with

$$F[\lambda\mathbf{x}, \mathbf{Y}, d\mathbf{Y}/(\lambda dx_1)...d\mathbf{Y}/(\lambda dx)_n, d^2\mathbf{Y}/(\lambda dx_1)^2.....d^m\mathbf{Y}/(\lambda dx_n)^m] = \lambda^pF(\mathbf{x}, \mathbf{Y}, d\mathbf{Y}/dx_1...d\mathbf{Y}/dx_n, d^2\mathbf{Y}/dx_1{}^2.....d^m\mathbf{Y}/dx_n{}^m) \tag{12}$$

admits, in the limit of weak dissipation, quasistationary homogeneous isotropic chaotic (turbulent) solutions which are practically independent of the nature of the instabilities or bifurcations (or even stirring forces) which have caused the chaotic state, the corresponding spectral density must be proportional to 1/f.

We note that the solution (7) leads to a weak (logarithmic) divergence of the integral over k in the last form of Eq. (6) and in Eq. (9). This seems to contradict the fact that in practical applications the autocorrelation

function A(t) is finite, and its value at t=0 is usually given in the problem at hand However, in practice one never deals with an infinite volume, and the physical wave-vectors are also limited. For instance in fluid dynamics, wave vectors exceeding the reciprocal average distance between neighboring fluid molecules correspond to thermal motions, and are therefore no longer meaningful for the hydrodynamic treatment. Due to its logarithmic character, the divergnce is thus without practical importance. Nevertheless, for a given level of chaos A(0), we can construct an approximate solution

$$u(k,z)=k^{r-n}v(z), \tag{13}$$

with $0<r<<1$, which avoids the divergence at $k=0$. To get the correct chaos level with $k<k_o$, k_o being an upper cutoff, we set

$$u=r[A(0)/v(0)]k_o^{-r}k^{r-n}v(z). \tag{14}$$

This yields for t=0 the result A(0) when we integrate over d^nk with an upper limit k_o. We notice that r is present both as a general factor, and as a small defect in the exponent of k. This is a general feature, present both in classical and quantum nonlinear systems with 1/f noise. In the limit $r\to 0$, the approximate solution tends to become exact. In the classical homogeneous, isotropic, turbulence theory, r can be arbitrarily small, while in the quantum 1/f theory (quantum electrodynamics), $r = \alpha A = (2\alpha/3\pi)(\Delta\mathbf{v}/c)^2 << 1/137$ is the well-defined infrared exponent of the process, with $\alpha=e^2/\hbar c=1/137$, as we know from the theory of infrared radiative corrections.

III. EXAMPLES

The general criterion developed in the preceeding section will now be illustrated on the basis of some examples.

III.1 Classical Turbulence Theory for the Current Carriers in Semiconductors

In the case of homogeneous, isotropic turbulence[4-6] caused in the electron-hole plasma of an infinite sample of a symmetric intrinsic semiconductor by dynamical instabilities of any kind, we start from the equations

$$\nu \mathbf{v}^{+} = (e/2c)\mathbf{v}^{-}\times\mathbf{B} - (1/n)\nabla P, \tag{15}$$

$$\nu \mathbf{v}^{-} = 2e[\mathbf{E} + \mathbf{v}^{+}\times\mathbf{B}/c] - (2/n)\nabla(P_p - P_n), \tag{16}$$

$$\nabla\cdot\mathbf{v}^{+} = 0 \qquad (n = \text{const}), \tag{17}$$

$$\nabla \times\mathbf{E} = -(1/c)\partial\mathbf{B}/\partial t, \tag{18}$$

$$\nabla \times\mathbf{B} = 2\pi e n\mathbf{v}^{-}/c, \tag{19}$$

$$\nabla\cdot\mathbf{B} = 0. \tag{20}$$

Here n is the total carrier concentration including an equal number of electrons and holes, ν/e their reciprocal mobility assumed to be the same for electrons and holes, P_n and P_p the partial pressures of electrons and holes, P the total carrier pressure, $2\mathbf{v}^{+}$, $\mathbf{v}^{-}$ the sum and the difference of the carrier drift velocities. Inertial terms proportional to the effective masses of the carriers, as well as electrostatic terms and compressibility terms have been neglected here in a consistent way[4-7], because we are interested in the low-frequency domain only. Although we do not work this out here, this system of equations can be shown to admit an energy theorem. Performing a Fourier expansion, we obtain

$$\nu \mathbf{v}^+(\mathbf{k}) = (e/2c)\sum_{\mathbf{k}'} \mathbf{v}^-(\mathbf{k}')\times \mathbf{B}(\mathbf{k}-\mathbf{k}') - (i/n)\mathbf{k}\cdot P(\mathbf{k}), \tag{21}$$

$$\nu \mathbf{v}^-(\mathbf{k}) = 2e[\mathbf{E}(\mathbf{k}) + \sum_{\mathbf{k}'} \mathbf{v}^+(\mathbf{k}')\times \mathbf{B}(\mathbf{k}-\mathbf{k}')/c] - (2i/n)\mathbf{k}(P_p - P_n), \tag{22}$$

$$\mathbf{k}\cdot \mathbf{v}^+(\mathbf{k}) = 0. \tag{23}$$

$$i\mathbf{k} \times \mathbf{E}(\mathbf{k}) = -(1/c)\partial \mathbf{B}(\mathbf{k})/\partial t, \tag{24}$$

$$i\mathbf{k} \times \mathbf{B}(\mathbf{k}) = 2\pi e n \mathbf{v}^-(\mathbf{k})/c, \tag{25}$$

$$\mathbf{k}\cdot \mathbf{B}(\mathbf{k}) = 0, \tag{26}$$

Substituting **E** from Eq. (22) into Eq. (24), we obtain with Eq. (25)

$$\partial \mathbf{B}(\mathbf{k})/\partial t + \mu k^2 \mathbf{B}(\mathbf{k}) = i\mathbf{k}\times \sum_{\mathbf{k}'} \mathbf{v}^+(\mathbf{k}') \times \mathbf{B}(\mathbf{k}-\mathbf{k}'), \tag{27}$$

where $\mu = c^2\nu/4\pi n e^2$. Eqs. (21) and (25) yield

$$\begin{aligned} \mathbf{v}^+(\mathbf{k}) = (i/4\pi\nu n) \sum_{\mathbf{k}''} \Big\{ &\mathbf{B}(\mathbf{k}'')[\mathbf{k}''\cdot \mathbf{B}(\mathbf{k}-\mathbf{k}'')] - \mathbf{k}''[\mathbf{B}(\mathbf{k}'')\cdot \mathbf{B}(\mathbf{k}-\mathbf{k}'')] \\ &-(\mathbf{k}/k^2)(1-\delta_{\mathbf{k},0})[\mathbf{k}''\cdot \mathbf{B}(\mathbf{k}-\mathbf{k}'')\ \mathbf{k}\cdot \mathbf{B}(\mathbf{k}'') - \mathbf{B}(\mathbf{k}'')\cdot \mathbf{B}(\mathbf{k}-\mathbf{k}'')\ (\mathbf{k}\cdot \mathbf{k}'')]\Big\}. \end{aligned} \tag{28}$$

Substituting this into Eq. (27), we obtain the fundamental dynamical field-equation of turbulence in the electron-hole plasma of a symmetric intrinsic semiconductor

$$\begin{aligned} \partial b_\beta(\mathbf{k},t)/\partial t + \mu k^2 b_\beta(\mathbf{k},t) = \sum_{\mathbf{k}'\mathbf{k}''} &b_j(\mathbf{k}-\mathbf{k}',t)\ b_l(\mathbf{k}'',t)\ b_m(\mathbf{k}'-\mathbf{k}'',t) \\ &\cdot(k_j\delta_{\beta s} - k_s\delta_{\beta j})[k_s\delta_{lm} - k_m''\delta_{ls} + (k_s'/k'^2)(1-\delta_{\mathbf{k}',0})(k_m''k_l' - \mathbf{k}'\cdot\mathbf{k}''\delta_{lm})], \end{aligned} \tag{29}$$

in terms of $\mathbf{b} \equiv \mathbf{B}/\sqrt{2\pi\nu n}$. This dynamical equation has the form of Eq. (3), with p=2 in Eq. (4), and with G defined as the r.h.s. minus the term in k^2 on the l.h.s.. Our sufficient criterion thus tells us that this nonlinear system will yield a 1/f spectrum. We present below the direct derivation for this example.

Multiplying Eq. (29) with $b_\alpha^*(\mathbf{k},t-\tau)$ and taking the average over a statistical ensemble which represents our notion of stationary turbulence, we obtain in quasi-stationary conditions

$$\partial w_{\alpha\beta}(\mathbf{k},\tau)/\partial\tau + \mu k^2 w_{\alpha\beta}(\mathbf{k},\tau)$$

$$= \sum_{\mathbf{k}'\mathbf{k}''} \langle b_\alpha^*(\mathbf{k},t-\tau) b_j(\mathbf{k}-\mathbf{k}',t) b_l(\mathbf{k}'',t) b_m(\mathbf{k}'-\mathbf{k}'',t)\rangle R_{jml\beta}, \qquad (30)$$

with $w_{\alpha\beta}(\mathbf{k},\tau) \equiv (L/2\pi)^3 \langle b_\alpha^*(\mathbf{k},t-\tau) b_\beta(\mathbf{k},t)\rangle$, where L is the edge of the cubic normalization box, and

$$R_{jml\beta} \equiv (k_j\delta_{\beta s} - k_s\delta_{\beta j})[k_s\delta_{lm} - k_m''\delta_{ls} + (k_s'/k'^2)(1-\delta_{\mathbf{k}',0})(k_m''k_l' - \mathbf{k}'\cdot\mathbf{k}''\delta_{lm})]. \qquad (31)$$

Multiplying Eq. (29) with more magnetic field components and averaging, we obtain equations connecting the fourth-order correlation tensor to the sixth-order tensor, and so on[4-7]. To end this infinite chain of equations for the correlation functions, we make a quasinormality assumption which expresses the fourth-order moment appearing in Eq. (30) according to the scheme

$$\langle ABCD\rangle = \langle AB\rangle\langle CD\rangle + \langle AC\rangle\langle BD\rangle + \langle AD\rangle\langle BC\rangle, \qquad (32)$$

valid if the four field components would approximate a joint normal distribution. This approximation does not alter the homogeneity of the system, which ultimately causes the 1/f spectrum. This approximation yields the closed equation

$$\partial w_{\alpha\beta}(\mathbf{k},\tau)/\partial\tau + \mu k^2 w_{\alpha\beta}(\mathbf{k},\tau)$$

$$= (2\pi/L)^3 w_{\alpha j}(\mathbf{k},\tau) \sum_{\mathbf{k}'} w_{lm}(\mathbf{k}',0) R_{jml\beta}(\mathbf{k},\mathbf{k}'). \qquad (33)$$

Isotropic turbulence requires $w_{\alpha\beta} = A_1(k)\delta_{\alpha\beta} + A_2(k)k_\alpha k_\beta$, with coefficients A_1 and A_2 related through Eq. (26), yielding

$$w_{\alpha\beta}(\mathbf{k},\tau) = (1/2)[\delta_{\alpha\beta} - k_\alpha k_\beta/k^2]u(k,\tau), \qquad (34)$$

where $u(k,\tau) = \Sigma_\alpha w_{\alpha\alpha}(\mathbf{k},\tau)$. Therefore, the scalar correlation function $u(k,\tau)$ satisfies the dynamical equation of homogeneous, isotropic, stationary turbulence

$$\frac{\partial v(k,x)}{\partial|x|} + v(k,x) = -\frac{1}{2}v(k,x)\int\frac{d^3k'}{k'^3}\frac{k^2+\mathbf{k}\cdot\mathbf{k}'}{(\mathbf{k}+\mathbf{k}')^2}\left[1-\left(\frac{\mathbf{k}\cdot\mathbf{k}'}{kk'}\right)^2\right]v(k',0), \tag{35}$$

where $v(k,x)\equiv k^{-3}u(k,\tau)$, and $x = \mu\tau k^2$ is a dimensionless variable replacing τ. We convince ourselves that the integral is independent of k, provided $v(k,x)$ does not depend on its first argument, by setting $\mathbf{k}'/k\equiv\kappa$. This yields a solution. However, with $v=e^{-m|x|}$ we get a logarithmic divergence at $\kappa=0$. We look for an exact solution of the form[4-7]

$$v(k,x) = hk^{\varepsilon}e^{-|x|m(k)}, \quad \text{or} \qquad u(k,x) = (h/k^{3-\varepsilon})\ e^{-|x|m(k)}, \tag{36}$$

where $m(k)$ is very close to a constant, almost independent of k, and h is a constant proportional to the intensity of the turbulence, or the turbulence level. Substituting this into Eq. (35) and performing the integration, we obtain a finite result only for $0<\varepsilon<2$:

$$m(k) = 1 + h\ r(\varepsilon)k^{\varepsilon}, \text{ with } r(\varepsilon)=[2\pi^2\cot\text{an}(\varepsilon\pi/2)]/[(1-\varepsilon^2)(3-\varepsilon^2)]. \tag{37}$$

We notice that $m(k)$ is indeed practically constant when $0<\varepsilon<<1$ is very small, arbitrarily small. The value $\varepsilon=0$ leads to a logarithmic divergence, but we can set $\varepsilon=0$ for practical purposes.

The spectral density corresponding to Eq. (36) with $\varepsilon=0$ is

$$w_{\alpha\beta}(\omega) = (1/\pi)\int_0^\infty \cos\omega\tau d\tau \int w_{\alpha\beta}(k,\tau)d^3k$$

$$= \frac{4}{3}\delta_{\alpha\beta}\int_0^\infty k^2dk\int_0^\infty d\tau\ u(k,\tau)\cos\omega\tau = \frac{4}{3}h\delta_{\alpha\beta}\int_0^\infty\frac{mkdk}{\omega^2+m^2k^2} = \frac{\pi}{3}\frac{h}{\omega}\delta_{\alpha\beta}. \tag{38}$$

This is a 1/f spectrum. At the low frequency end we do not get a divergent spectral integral, because the more exact form of the spectrum with a finite small $\varepsilon<<1$ is[4-7]

$$\int_0^\infty \frac{mk^{1+\varepsilon}dk}{\omega^2+m^2k^4} = \frac{1}{\omega^{1-\varepsilon/2}m^{\varepsilon/2}}\int_0^\infty \frac{x^{1+\varepsilon}dx}{1+x^4}, \tag{39}$$

which is proportional to $f^{\varepsilon/2-1}$. It is interesting to note that for $\varepsilon<<1$ $\cot \varepsilon\pi/2 \approx 2/\varepsilon\pi$ in Eq. (37), and that the value of ε calculated from Eq. (37) is therefore proportional to h, or to the intensity of the turbulence. This feature of the classical theory[4-7] is expressed with fascinating clarity in the quantum form of the theory, where ε is replaced by $2\alpha A$ which also appears as a intensity factor multiplying the quantum 1/f noise.

The essential element which led to the 1/f spectrum in the classical turbulence theory is the *nonlinearity* of the equations of motion, caused by the reaction of the electric currents back on themselves via the generated electromagnetic field. The same feedback reaction, via the electromagnetic field, also caused the nonlinearity in the quantum 1/f theory, and in QED in general, leading in the same way to an identical 1/f spectrum, this time with a physically more meaningful $\varepsilon=2\alpha A$. This *nonlinearity* induces the coupling between various scales of turbulence and leads to the dynamical equilibrium between eddies of all sizes, expressed by the 1/f spectrum. In the $\varepsilon=0$, or $\alpha A=0$, limit, this dynamical equilibrium assumes both for the quantum case and for homogeneous, isotropic, turbulence in the unbounded semiconductor sample the simplest form, characterized by scale-homogeneity, or scale invariance. Indeed, replacing for $\varepsilon=0$ in Eqs. (35) and (36) k and k' by λk and $\lambda k'$, while also replacing τ by τ/λ^2, (or ω by $\lambda^2\omega$), Eq. (35) is not affected, and λ drops out. We conclude that *in the weak turbulence limit ($\varepsilon=0$) we obtain perfect self-similarity of the turbulence process at all scales in space and time, classically and quantum-mechanically*. The implied scale invariance is caused by the absence of any characteristic length or time scale, or by the presence of a sliding scale. Indeed, the frequency scale μk^2 is a function

of the size of the eddies, given by the wave number k which can have any value. The actual frequency and wave-number spectra are closely related fractals, but in the weak-turbulence limit they approach an exact 1/f and $1/k^3$ spectrum respectively. In fact, we are here understanding the nonlinear dynamics which shapes this fractal for the first time.

III.2 Turbulence Theory for Drude Electrons in Metals

Our classical turbulence theory can be extended to the case of metals or degenerate extrinsic semiconductors[7] in the Drude model. The system of integro-differential equations is quite different,

$$\nu \mathbf{v}^{+} = -e\mathbf{E} - (e/c)\mathbf{v}\times\mathbf{B} - (1/n)\nabla P \tag{40}$$

$$\nabla\times\mathbf{E} = -(1/c)\partial\mathbf{B}/\partial t \tag{41}$$

$$\nabla\times\mathbf{B} = -(4\pi en/c)\mathbf{v} \tag{42}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{43}$$

and leads to a third-order nonlinearity[7] in the resulting closed equation of motion, or nonlinear field-equation, which replaces Eq. (28):

$$\partial\mathbf{B}(k,t)/\partial t + \nu k^2\mathbf{B}(\mathbf{k},t) = -(c/4\pi ne)\mathbf{k}\times\int d^3k'\mathbf{B}(\mathbf{k}-\mathbf{k}',t)\times[\mathbf{k}'\times B(\mathbf{k}',t)]. \tag{44}$$

This is again in the form of Eq. (3), with p=2 in Eq. (4). We thus expecta 1/f spectrum in this system as well. This time we only sketch the derivation. The corresponding infinite chain of equations for the correlation tensors now goes in steps of one. As was shown above, it went in steps of two for semiconductors. The third-order correlation can be eliminated between the first and second equations in the chain. The resulting dynamical equation[7] for homogeneous, isotropic, stationary turbulence, which replaces Eq. (35), with the same notations, using $\mathbf{e}_3$ as the unit vector of the third axis, is

$$- \frac{\partial^2 v(k,x)}{\partial x^2} + v(k,x)$$
$$= - \frac{1}{2}\int \frac{d^3\kappa}{\kappa^3} \frac{1+\kappa\cdot \mathbf{e}_3}{|\mathbf{e}_3+\kappa|^3}\left[1 - \frac{(\kappa\cdot \mathbf{e}_3)^2}{\kappa^2}\right] v(\kappa k, \kappa^2 x) v(k|\mathbf{e}_3+\kappa|, x|\mathbf{e}_3+\kappa|^2). \qquad (45)$$

This also admits, in the $\varepsilon=0$ limit of weak turbulence, a solution $v(k,x)$ which does not depend on the first argument, and $u(k,x) = k^{-3}e^{-xm(x)}$, this time with an x-dependent m. With the change of variables $t=\omega\tau$ and $k'=k/\sqrt{\omega}$ in the second (middle, involving u) form of Eq. (38), x remains invariant, and a factor $1/\omega$ will appear in front of the integrals which themselves will just yield a constant factor independent of ω, τ or k. We thus obtain again a universal 1/f spectrum. As is shown in detail elsewhere[7], this 1/f spectrum is expressed in the corresponding current and voltage fluctuations which can be observed in the semiconductor or metallic medium. We conclude that the 1/f spectrum is a general property of electrically conducting systems in interaction with the electromagnetic field, a property which is caused by the nonlinearity of the system of carriers and field in mutual interaction due to the absence of a characteristic scale in the nonlinear equation of motion, and which finds its clearest expression in the Quantum 1/f Effect.

III.3 Theory of Highway Traffic Fluctuations

Musha and Higuchi[8,9] discovered the 1/f spectrum of highway traffic fluctuations empirically in 1977, also developing a model based on a postulated linear dependence of the average traffic speed v on the linear concentration of cars n(x) on the road, $v=v_0(1-n/n_s)$. The model was recognized to be similar to Burger's model of turbulence, and was simulated numerically leading to a 1/f-like spectrum at low frequencies.

The present paper develops a statistical turbulence theory for the Musha model, showing how in the low wave number and low frequency limit a universal 1/f spectrum emerges. In this limit, this is a particular case of the author's sliding-scale-invariant class of nonlinear systems, all characterized by the presence of a universal 1/f spectrum. The author's classical[10,11] and quantum[12] 1/f theory is another example in the same class[13].

Musha writes the traffic current J as a sum of drift and diffusion currents

$$J = nv - D\partial n/\partial x = nv_o - (v_o/n_s)n^2 - D\partial n/\partial x, \tag{46}$$

where D is a "diffusion coefficient". The equation of continuity is[8,9]

$$0 = \partial n/\partial t + \partial J/\partial x \equiv (\partial/\partial t + v_o\partial/\partial x)n - 2(v_o/n_s)n\partial n/\partial x - D\partial^2 n/\partial x^2. \tag{47}$$

This is Musha's fundamental equation of traffic dynamics, which also was written[8] in a system of reference defined by

$$x' = -x + v_o t, \qquad t' = t \tag{48}$$

in the final form[8]

$$\partial n/\partial t' + an\partial n/\partial x' = D\partial^2 n/\partial x'^2, \tag{49}$$

where $a \equiv 2v_o/n_s$.

We normalize the concentration to $n_s/2$, the coordinate along the road x to D/v_o, and the time to D/v_o^2, thereby obtaining the dimensionless form of the fundamental traffic-dynamical equation

$$\partial n(x,t)/\partial t + n(x,t)\partial n(x,t)/\partial x = \partial^2 n(x,t)/\partial x^2, \tag{50}$$

where we did not bother to change the notation, and returned to the original unprimed notation. Expanding in a Fourier series over the interval L, we obtain

$$\partial n/\partial t + i\Sigma_{k'}k'n(k')n(k-k') = -k^2 n(k,t). \tag{51}$$

Comparing Eq. (50) with Eqs. (1),(2), or Eq. (51) with Eqs. (3),(4), we notice that this time our criterion is not satisfied, due to the r.h.s. term. However, in the limit of small k that term becomes negligible, and thus, our criterion becomes applicable, and we should get a 1/f spectrum in the

low frequency limit. This would correspond to p=1 in Eqs. (2) and (3). We derive this 1/f spectrum below.

Defining the autocorrelation function

$$A(\xi,\tau) \equiv \langle n^*(x,t)n(x+\xi,t+\tau)\rangle = \Sigma_k \langle n^*(k,t)n(k,t+\tau)\rangle e^{ik\xi} = \Sigma_k U(k,\tau)e^{ik\xi}, \quad (52)$$

as a turbulent ensemble average $\langle\ldots\rangle$, in quasistationary homogeneous conditions we obtain by multiplication of Eq. (51) with $n^*(k, t-\tau)$, after ensemble averaging,

$$\partial U/\partial\tau + k^2U(k,\tau) + i\Sigma_{k'}k'\langle n^*(k, t-\tau)n(k',t)n(k-k',t)\rangle = 0. \quad (53)$$

Linking the complex function $U(k,\tau)$ to a third-order correlation function, this is the first equation of an infinite chain of equations connecting the N^{th} order to the order N+1 correlation function. Applying the operator $-\partial/\partial\tau + k^2$ to Eq. (53), and using the complex conjugate of Eq. (51) to define the action of this operator on the first factor inside the averaging brackets in Eq. (53), we obtain

$$-\partial^2U/\partial\tau^2 + k^4U = -i\Sigma_{k'}k'\langle[i\Sigma_{k''}k''n^*(k'',t-\tau)n^*(k-k'',t-\tau)]n(k',t)n(k-k',t)\rangle$$
$$\approx \Sigma_{k',k''}k'k''[U(k'',0)U(k',0)\delta_{k,0} + U(k',\tau)U(k-k',\tau)\delta_{k''k'} + U(k',\tau)U(k-k',\tau)\delta_{k'',k-k'}]. \quad (54)$$

The last form was obtained by approximating the fourth-order correlation function with its expression in terms of the second-order correlation applicable for Gaussian processes:

$$\langle ABCD\rangle = \langle AB\rangle\langle CD\rangle + \langle AC\rangle\langle BD\rangle + \langle AD\rangle\langle BC\rangle. \quad (55)$$

This approximation was used by Heisenberg[14] in his turbulence theory, and was successfully verified in its practical applicability by Uberoi[15] and Batchelor[16].

In the limit $L\to\infty$, setting $U(k,\tau)L/2\pi = u(k\tau)$, we write for $k\neq0$ Eq. (54) in the form

$$-\partial^2u/\partial\tau^2 + k^4u = \int[k'^2u(k',\tau)u(k-k', \tau) + k'(k-k')u(k',\tau)u(k-k',\tau)]dk'$$

or

$$-\partial^2 u(k,\tau)/\partial\tau^2 + k^4 u(k,\tau) = k\int k' u(k',\tau) u(k-k',\ \tau) dk'. \tag{56}$$

This is our fundamental dynamical equation of traffic turbulence. All unspecified limits on integrals are from $-\infty$ to ∞, as we mentioned earlier. From the symmetry $A(\xi,\tau) = A(-\xi,-\tau) = A^*(\xi,\tau)$ we see that a physically acceptable solution of this equation must satisfy the conditions

$$u(k,\tau) = u(-k,-\tau) = u^*(-k,-\tau) = u^*(k,-\tau). \tag{57}$$

A solution of the form

$$u(k,\tau) = V(k)e^{-imk\tau},\ \text{with } V(k) = V(-k) = V^*(k) \tag{58}$$

where m is a real constant, substituted into Eq. (56), yields

$$k(m^2 + k^2)V(k) = \int k' V(k') V(k-k') dk'. \tag{59}$$

In the low wave number region $k<<1$ (i.e., $k<<v_o/D$) we neglect the k^2 term which arises from diffusion, and get

$$m^2 k V(k) = \int k' V(k') V(k-k') dk'. \tag{60}$$

A solution $V(k) = C|k|^{\varepsilon-1}$, with arbitrarily small $\varepsilon>0$, yields the value of C, independent of k only in the limit of $\varepsilon\rightarrow 0$:

$$m^2/C = \int_{-\infty}^{\infty} k'|k'|^{\varepsilon-1}|1-k'/k|^{\varepsilon-1} dk'/k \ = |k|^{\varepsilon} \int_{-\infty}^{\infty} \kappa|\kappa|^{\varepsilon-1}|1-\kappa|^{\varepsilon-1} d\kappa$$
$$= |k|^{\varepsilon} \int_0^2 |\kappa|^{\varepsilon}|1-\kappa|^{\varepsilon-1} d\kappa \approx 2/\varepsilon = I \approx \text{const}, \tag{61}$$

where we used the substitution $k'/k=\kappa$. The divergence at $\varepsilon=0$ disappears when we return to a finite value of the maximal road length L, which corresponds to a minimal k (or k') value $k_0=2\pi/L$, and transforms the integrals back into sums.

Eq. (61) establishes a proportionality between the level of the turbulence,described by C, and the magnitude of the small parameter ε as in earlier turbulence calculations[10,11]. The same fundamental feature is more clearly expressed, without the pseudo-singularities present here, in the quantum 1/f theory[12], where ε becomes the infrared exponent αA known from quantum electrodynamics. Indeed, αA is present there, just as we see it come in here, both as a factor in front of the final result, and as a small defect in the exponent of the frequency. We also mention that the apparent absolute determination of C by Eq. (61) is just an artifact which reminds us that we omitted a source term in Eqs. (47), (49-51), (53-54), (56), which comes in as a δ function of time, $h(k)\delta(t)$ on the r.h.s. of Eq. (53), only if we assume the excitation in Eq. (47) to depend on time like white noise. Once that source term is written explicitly, it will allow us to determine C as this was done earlier[11] for a different equation, and it does not affect our equations for $\tau \neq 0$.

We can rewrite Eq. (52) in the form

$$A(\xi,\tau) \equiv \langle n^*(x,t)n(x+\xi,t+\tau)\rangle = \Sigma_k U(k,\tau)e^{ik\xi} \equiv \int dk\ u(k,\tau)e^{ik\xi}$$
$$= C\int e^{ik(\xi-m\tau)}|k|^{\varepsilon-1}dk = |\xi-m\tau|^{-\varepsilon}(m^2/I)\int e^{i\kappa}|\kappa|^{\varepsilon-1}d\kappa \approx m^2|\xi-m\tau|^{-\varepsilon}. \quad (62)$$

Here we have used the substitution $k|\xi-m\tau|=\kappa$, and we have taken the limit $\varepsilon\to 0$ in the integral and in I, noting that they exhibit the same divergence, and therefore can be simplified in the limit. The cancellation of the divergences in the expression of the autocorrelation function shows that this important function is finite even in the continuum limit. It is only the spectral distribution of turbulence which exhibits a singularity at low wave numbers and frequencies.

Since only the limit $\varepsilon=0$ satisfies the fundamental dynamical equation of traffic turbulence, Eq. (62) means that the autocorrelation function is a constant. It also indicates that the constant has to be interpreted as the limit of a slowly decreasing power law which depends

only on $|\xi-\tau|$; therefore the spectrum in wave numbers is the same as the spectrum in frequencies. The Fourier transform with respect to τ is the spectral density

$$S_n'(\xi,\omega) = \int \langle n^*(x,t)n(x+\xi,t+\tau)\rangle e^{i\omega\tau}d\tau = C\iint (dk/|k|^{1-\varepsilon})e^{ik(\xi-m\tau)+i\omega\tau}d\tau. \qquad (63)$$

To leave the moving (primed) frame and return to the system of reference at rest, we replace ω by $\omega\text{-}kv_o$ in the last integrand, actually just by $\omega\text{-}k$ in our (v_o=1) normalization:

$$S_n(\xi,\omega) = \int \langle n^*(x,t)n(x+\xi,t+\tau)\rangle e^{i(\omega-kv_o)\tau}d\tau$$
$$= C\iint (dk/k^{1-\varepsilon})e^{ik(\xi-m\tau)+i(\omega-k)\tau}d\tau. \qquad (64)$$

The spectral density of concentration fluctuations in a given point is obtained by setting ξ=0. The integration with respect to τ yields a delta function $2\pi\delta[\omega-(m+1)k]$, and we finally obtain the spectrum

$$S_n(0,\omega) = C[(m+1)/\omega]^{1-\varepsilon} \qquad (65)$$

where again the constant will turn out to be finite when we coarse grain the integrals. As mentioned above, the constant will actually be determined by the excitation term omitted in Eq. (56).

According to Eq. (2) we can write $J(k,\omega) = (\omega/k)n(k,\omega)$ in the system at rest. Therefore, for J we include a factor $(\omega/k)^2$ into the integrand of Eq. (64), and we get again the spectrum

$$S_J(\xi,\omega) = \int \langle J^*(x,t)J(x+\xi,t+\tau)\rangle e^{i(\omega-kv_o)\tau}d\tau$$
$$= C\iint (\omega^2 dk/k^{3-\varepsilon})e^{ik(\xi-m\tau)+i(\omega-k)\tau}d\tau; \qquad (66)$$
$$S_J(0,\omega) = C(m+1)^2\ [(m+1)/\omega]^{1-\varepsilon} \approx \mathrm{Const}/\omega. \qquad (67)$$

Due to the neglect of the k^2 term in Eq. (59), the universal 1/f spectrum will be limited towards high frequencies by $\omega < v_o^2/D$, and the 1/k-spectrum by $k < v_o/D$. This limitation occurs because only Eq. (60) satisfies the condition of being free of any characteristic scale, thus

exhibiting our sliding scale invariance[13], while Eq. (59) does not have this property. We have thus constructed a statistical dynamic theory of traffic turbulence, proving analytically Musha's earlier result, without any pretension of mathematical rigor. Traffic turbulence arising from instabilities of the laminar traffic flow can be considered as a form of classical fluid-dynamical chaos.

III.4 Quantum 1/f Noise (QED)

The nonlinearity causing the 1/f spectrum of turbulence in both semiconductors and metals is the reaction of the field generated by charged particles and their currents back on themselves. The same nonlinearity is present in quantum electrodynamics (QED), causes the infrared divergence, infrared radiative corrections for cross sections and process rates, and causes the quantum 1/f effect. The basic equations of QED are

$$\partial_\mu \partial^\mu A^\nu = J^\nu, \qquad J^\nu = -ie\psi\gamma^\nu\psi, \tag{68}$$

$$(\partial_\mu \gamma^\mu + m)\psi = -ieA^\nu\gamma_\nu\psi \tag{69}$$

Here ∂_μ is a shorthand for $\partial/\partial x_\mu$, γ^μ are the Dirac matrices, the Greek indices run from 0 to 3, and ψ is the Dirac spinor field.

We can formally solve Eq. (68) for the potential A^ν in momentum space (i.e. by Fourier transforming)

$$A^\nu = (1/\partial^\mu \partial_\mu) J^\nu, \tag{70}$$

which can be substituted into Eq. (70), yielding with the expression (68) of the current J^ν,

$$(\partial_\mu \gamma^\mu + mc/\hbar)\psi = -\alpha[(1/\partial^\mu\partial_\mu)\psi\gamma^\nu\psi]\gamma_\nu\psi. \tag{71}$$

The integral operator $(1/\partial^\mu\partial_\mu)$ has a kernel $1/q^2$, where q is the Fourier four-vector corresponding to x. Eq. (71) contains a third-order nonlinearity and is therefore similar to the dynamical equation (29) describing turbulence in semiconductors. In the spinless non-relativistic limit, Schrödinger's equation yields

$$\partial\psi/\partial t = [-i\hbar\nabla + A]^2\psi, \tag{72}$$

or, with Eq. (68),

$$2mi\hbar\partial\psi/\partial t = [-i\hbar\nabla - i(e^2\hbar/mc^2)(1/\partial^\mu\partial_\mu)\psi^*\nabla\psi]^2\psi \tag{73}$$

At very low frequencies the last term in rectangular brackets is dominant on the r.h.s., leading to

$$2i\partial\psi/\partial t = -\hbar[(e^2/mc^2)(1/\partial^\mu\partial_\mu)\psi^*\nabla\psi]^2\psi \tag{74}$$

Both Eq. (71), in which we neglect at low wave-numbers the terms in $\partial/\partial x_j$ (j=1,2,3) and eliminate the term with the rest mass m through a substitution $\psi=\phi e^{im\gamma t}$, and right away Eq. (74), satisfy our criterion with p=2 in Eq. (2). Therefore, we expect a 1/f spectrum of current fluctuations, i.e., of cross sections and process rates in physics. This is in agreement with the well-known, and experimentally verified, results of the conventional Quantum 1/f Theory.

In conclusion, we realize that, both in classical and quantum mechanical nonlinear systems, the limiting behaviour at low wave numbers is usually expressed by homogeneous functional dependences, (as shown in Eq. 2), leading to fundamental 1/f spectra on the basis of our criterion.

IV. ANALYTICAL CALCULATION OF MOBILITY FLUCTUATIONS IN SEMICONDUCTORS, BASED ON THE QUANTUM 1/f CROSS-CORRELATION FORMULA

Together with the graduate student Thomas H. Chung, the author has performed an analytical calculation of mobility fluctuations in silicon and gallium arsenide, using the new quantum 1/f cross-correlations formula derived by the author in the previous AFOSR Grant period, and included in the July 1989 Final Technical Report. This calculation is of mjor importance for the 1/f noise-related optimization both of the two types

of materials, and of the many devices constructed with them for military and civilian applications in the electronic and opto-electronic industry.

The new cross-correlation formula gives the cross-spectral density which describes the way in which symultaneous quantum 1/f scattering rate fluctuations ΔW obseved in the direction of the outgoing scattered wave-vector **K'** are correlated with those in the **K"** direction, when the two corresponding incoming current carriers have the wave vectors $\mathbf{K}_1$ and $\mathbf{K}_2$:

$$S_{\Delta W}(\mathbf{K}_1,\mathbf{K}';\mathbf{K}_2,\mathbf{K}'';f) = (2\alpha/3\pi f)(\hbar/m^*c)^2 W_{\mathbf{K}_1,\mathbf{K}'} W_{\mathbf{K}_2,\mathbf{K}''}[(\mathbf{K}'-\mathbf{K}_1)^2+(\mathbf{K}''-\mathbf{K}_2)^2]\delta_{K_1,K_2}. \quad (75)$$

The form conjectured by us earlier had $2(\mathbf{K}'-\mathbf{K}_1)(\mathbf{K}''-\mathbf{K}_2)$ in place of the rectangular bracket.

IV.1 Impurity Scattering

For impurity scattering of electrons in solids, fluctuations $\Delta\tau$ of the collision times τ will cause mobility fluctuations

$$\Delta\mu_{band}(t) = [e/m^*\langle\langle v^2\rangle\rangle]\Sigma_{\mathbf{K}} v_{\mathbf{K}}^2 \Delta\tau(t) n_{\mathbf{K}}, \quad (76)$$

where $\langle\langle v^2\rangle\rangle$ is both the average over all states of wave-vectors **K**, with occupation numbers $n_{\mathbf{K}}$, in the conduction band, and the thermal equilibrium average of the quadratic carrier velocities. With the help of the relation

$$1/\tau(\mathbf{K}) = (V/8\pi^3)\int(1 - \cos\theta'/\cos\theta) W_{\mathbf{K},\mathbf{K}'} d^3K', \quad (77)$$

the mobility fluctuations are reduced to fluctuations of the elementary scattering rates $W_{\mathbf{K},\mathbf{K}'}$, governed by Eq. (75). Here V is the volume of the normalization box which disappears in the final result, θ and θ' respectively the angles **K** and **K'** form with the direction of the applied field. One finally obtains after tedious multiple integrations

$$\mu^{-2}S_{\Delta\mu}(f) = [256\pi\alpha\kappa^2\varepsilon^4\hbar^{12}/3m^{*8}Z^4e^8N_i^2](1/f)\Sigma_K K^{10}[\ln(1+a^2)-a^2/(1+a^2)]^{-3}[(2a^2+a^4)/(1+a^2)-2\ln(1+a^2)]F(E_K)[\Sigma_K v_K^2\tau(K)F(E_K)]^{-2}, \quad (78)$$

where $a=2K/\kappa$, $\kappa^2=e^2n(T)/\varepsilon k_BT$, $n(T)$ is the electron concentration, $F(E_K)=\exp(E_F-E_K)$ for non-degenerate semiconductors, N_i the concentration of impurities of charge Ze and ε the dielectric constant. The corresponding partial Hooge parameter for impurity scattering is thus

$$\alpha_i = [4\sqrt{2}\pi\alpha\kappa\hbar^5N_c/3m^{*7/2}(k_BT)^{3/2}c^2]\int_0^\infty dx\,x^{11/2}e^{-x}$$
$$[\ln(bx+1)-bx/(bx+1)]^{-3}[(2bx+b^2x^2)/(bx+1)-2\ln(bx+1)]$$
$$\left\{\int_0^\infty dx\,x^3e^{-x}[\ln(bx+1)-bx/(bx+1)]^{-1}\right\}^{-2}. \quad (79)$$

IV.2 Acoustic Electron-Phonon Scattering

In this case the calculation is similar, and leads to the result

$$\alpha_{ac} = [32\pi\alpha N_cm^*C^7\hbar^3/3c^2k_BT)^4]\Big\{(1/R^2)\int_1^\infty dx\,x^{-4}$$
$$[(x-1)^7/7+(R+1)(x-1)^6/6+R(x-1)^5/5]$$
$$[(x-1)^5/5+(R+1)(x-1)^4/4+R(x-1)^3/3]\exp(-x^2/4R)$$
$$+\int_0^1 dx\,x^{-4}[(x+1)^5/5-(x+1)^6/6+(x-1)^5/5+(x-1)^6/6]$$
$$[(x+1)^3/3+(x-1)^4/4+(x-1)^3/3-(x+1)^4/4]\exp(-x^2/4R)$$
$$+\int_1^\infty dx\,x^{-4}[(x+1)^5/5-(x+1)^6/6][(x+1)^3/3-(x+1)^4/4]\exp(-x^2/4R)\Big\}, \quad (80)$$

where $R=k_BT/2m^*C_1^2$, C_1 is the deformation potential, and N_c is the effective density of states for the conduction band.

IV.3 Non-Polar Optical Phonon Scattering

This time one obtains

$$\alpha_{n.o.ph} = [8\pi\sqrt{2\hbar\omega_0}\alpha N_c\hbar^2/3m^{*5/2}c^2\omega_o]\{\int_0^\infty dx x^{5/2}$$
$$[(F+1)(x-1)^{1/2}\theta(x-1)+F(x+1)^{1/2}]^{-4}$$
$$[(F+1)^2(x-1)(2x-1)\theta(x-1)+F^2(x+1)(2x+1)]\exp(-\hbar\omega_o x/k_BT)\}$$
$$\int_0^\infty dx x^{3/2}[(F+1)(x-1)^{1/2}\theta(x-1)+F(x+1)^{1/2}]^{-1}\exp(-\hbar\omega_o x/k_BT)\}^{-2}, \qquad (81)$$

where $F=[\exp(\hbar\omega_o/k_BT)-1]^{-1}$, and ω_o is the optical phonon frequency.

V. CALCULATION OF COLLECTOR 1/f NOISE IN BJTs

Based on a consequent application of the conventional quantum 1/f approach, we show that for the calculation of conventional quantum 1/f noise in the collector current of BJTs the effective lifetime τ of minority carriers in the base must be used instead of their diffusion time τ_D through the base. This allows us to represent the effective number of carriers in the denominator of our quantum 1/f formula as $I\tau/e$ for noise in the collector current, and the theory of Van der Ziel and Kleinpenning remains valid with τ_D replaced by τ. A long-standing difficulty present in the application of bulk noise sources to the collector current is thus removed in a natural way, and the agreement of the quantum 1/f theory with the experiment is substantially improved, particularly for narrow base BJTs.

1/f noise in bipolar junction transistors (BJTs) was elegantly treated by van der Ziel[17-19] who applied a Hooge-type approach similar to Kleinpenning's treatment[20] of pn junctions, and used experimental data to determine the Hooge constant which was in turn compared with the quantum 1/f theory. However, since the BJT is a minority carrier device, it requires the application of the quantum 1/f (Handel) equation[19,1-3,21]

from the beginning, for the correct interpretation of the number of carriers in the denominator of the Langevin noise source.

In the most elementary model[22] of a BJT, the collector current I_C arises from minority carriers injected from the emitter into the base, which diffuse across the width X_B base and are then all swept across the reverse-biased collector junction by the built-in field of the junction. If we neglect the usually small leakage current of the collector junction and the small fraction of the carriers recombining in the base, we get for a n^+pn BJT

$$I_C = AqD_n[n_{oB}\exp(qV_{BE}/kT)/X_B], \tag{82}$$

where A is the cross sectional area of the base, q=-e is the charge of the minority carriers in the base, D_n their diffusion coefficient in the base, $n_B(0)=n_{oB}\exp(qV_{BE}/kT)$ is the electron concentration at the limit of the emitter space charge region, V_{BE} is the applied base - emitter voltage, and X_B is the width of the base. The expression in rectangular brackets is the electron concentration gradient calculated with the boundary condition of a vanishing electron concentration at the limit of the collector space charge region. We assume the base to be much narrower than the electron diffusion length $L_n=(D_n\tau)$, $X_B<<L_n$, but sufficiently wide to avoid ballistic electron transport across the base. Usually X_B is a fraction of a micron.

Quantum 1/f fluctuations of the collisional cross sections of the electrons in the base will yield fluctuations of the diffusion constant, and of the mobility $(\delta D_n/D_n=\delta\mu/\mu)$

$$\delta I_C = Aq(\delta D_n)[n_{oB}\exp(qV_{BE}/kT)/X_B]. \tag{83}$$

The corresponding spectral density of fractional fluctuations $I^{-2}S_{I_C}$ is

$$I_C^{-2}<(\delta I_C)^2>_f = D_n^{-2}<(\delta D_n)^2> = \mu^{-2}<(\delta\mu)^2> = \alpha_n/fN. \tag{84}$$

In the last step our quantum 1/f equation[19,1-3] was used, where N is the number of carriers which define the scattered, or diffused, current leaving the base and emerging in the collector, while $\alpha_n = \alpha A_n$ is the effective quantum 1/f noise coefficient, or Hooge constant. The number of

electrons N is thus determined by the effective lifetime τ of the electrons, which will be slightly lower than the lifetime in the unbounded collector material, due to the collector lead contact processes, and due to lateral surface recombination. Indeed, we can write $N = \tau I_C/q$. Thus we finally obtain the spectral density of the collector current fluctuations

$$S_{Ic} = \alpha_n I q/f\tau, \tag{85}$$

in which τ is the effective lifetime of the majority carriers in the collector. This expression is simpler, but similar to the expression derived earlier, with the important difference that now we have a lifetime of the carriers in the denominator, while before it was the usually much smaller diffusion time $\tau_D = X_B^2/D_n$ of the electrons in the base. Eq. (85) also implies that in narrow-base BJTs of various base-widths α_n will be constant, as in other devices, rather than α_n/τ_D.

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